SAT

- Given a set **C** of clauses over a set **P** of propositional variables
  - a propositional variable can be assigned *true* or *false*
  - a *literal* is a variable or its negation
  - a *clause* is a disjunction of literals
- Is there a truth assignment for **P** that satisfies all clauses in **C**

- SAT is NP-complete so…
  - $\Rightarrow$ Must give up something to accept acceptable behavior?
  - $\Rightarrow$ Worst-case analysis irrelevant to AI? What is the average case complexity?

Average-case analysis

- Earlier empirical work by Goldberg suggests that SAT is readily solvable “on average” in polynomial time
- Need a distribution of problems for “average case” complexity
  - $\Rightarrow$ Goldberg’s distribution has a preponderance of easy problems

Constant density model

- *Random P-SAT*
  - *M* clauses over *N* variables
- Each clause is generated by including a variable in a clause with some probability *P*, and negating it with probability 0.5
  - to avoid trivially satisfiable/unsatisfiable theories, empty and unit clauses are disallowed

- Analytic and empirical evidence suggests that most problems drawn from this distribution are computationally easy

Fixed clause length model

- *Random K-SAT*
  - M clauses over N variables
  - each clause has exactly K literals
- Each clause is generated by
  - randomly pick K distinct variables from the N variables
  - negate each with probability 0.5

Davis-Putnam procedure

function DP(Σ , **P**)

Unit propagate Σ

if a contradiction is discovered **then return** *false*

else if all variables are valued **then return** *true*

else

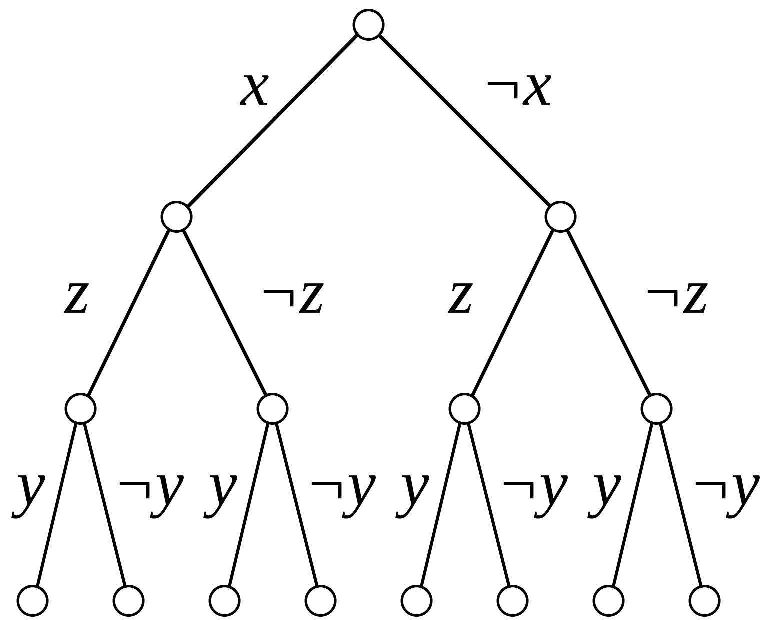
Let x be some unvalued variable

return DP($\Sigma \cup \{x\}$, **P**) **or** DP($\Sigma \cup \{\neg x\}$, **P**)

endif

Example

$\mathbf{P} = \{x, y, z\}$ $\Sigma = \{ \{\neg x, y, z\}, \{\neg x, \neg y, z\}, \{\neg x, \neg z\} \}$



DP calls for Random 3-SAT

- Figure 2 from Selman et al

Factored data

- Figure 3 from Selman et al

Phase transitions

- Figure 4 from Selman et al